Demographics, Wealth, and Global Imbalances in the Twenty-First Century

Adrien Auclert, Hannes Malmberg, Frédéric Martenet and Matthew Rognlie

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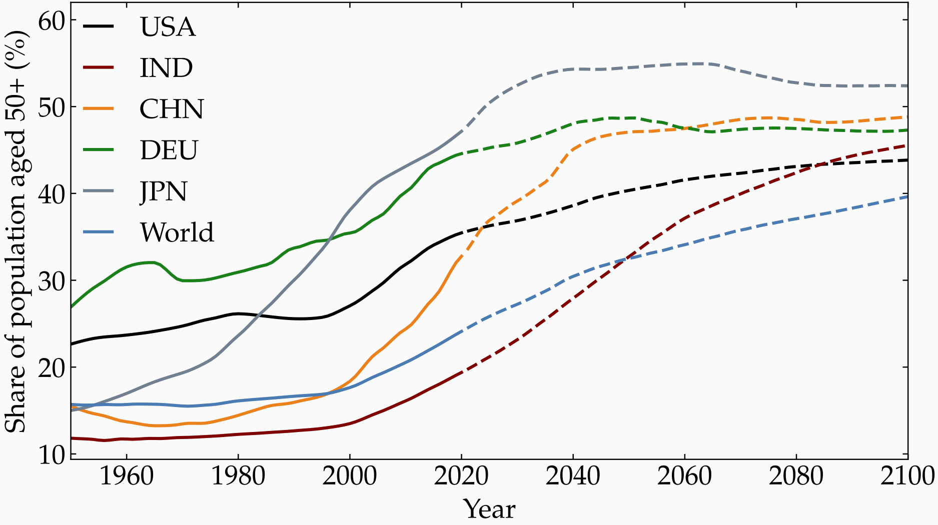
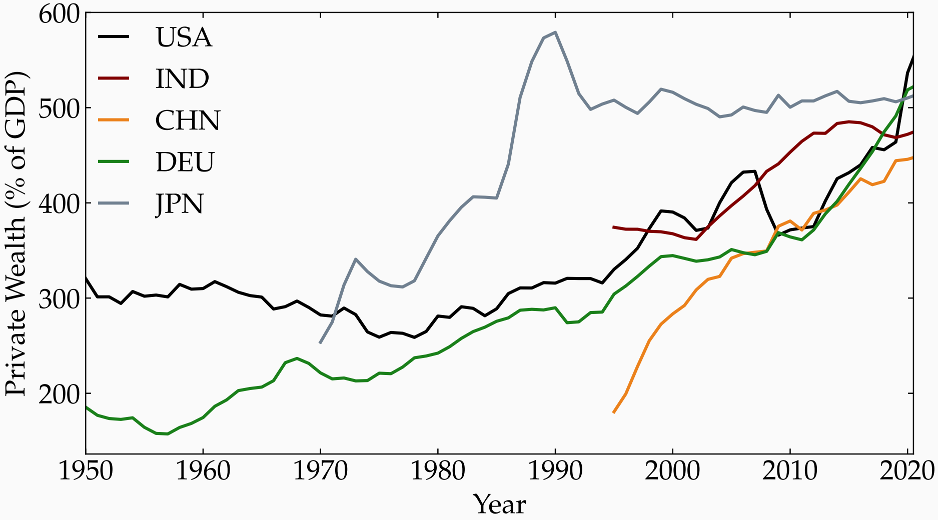
The world population is aging...



Source: 2022 United Nations World Population Prospects

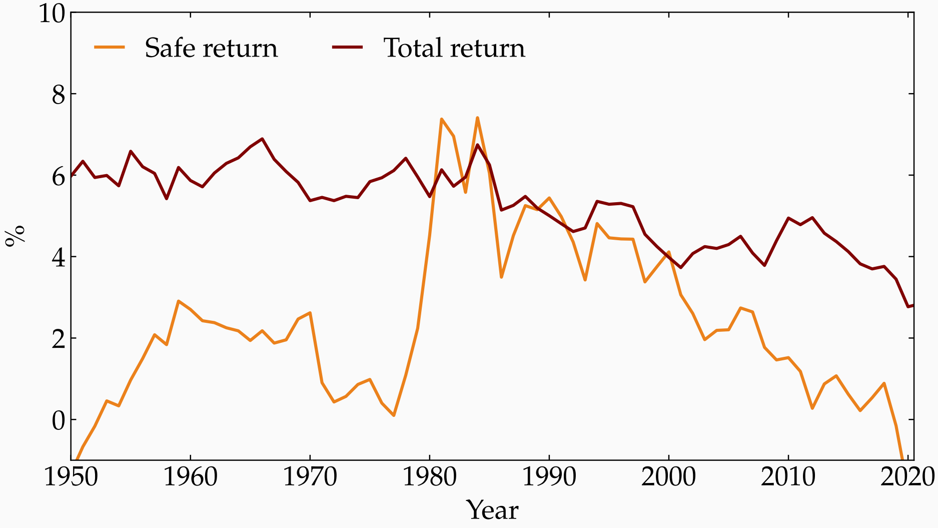
…wealth-to-GDP ratios are increasing…



Source: World Inequality Database (WID)

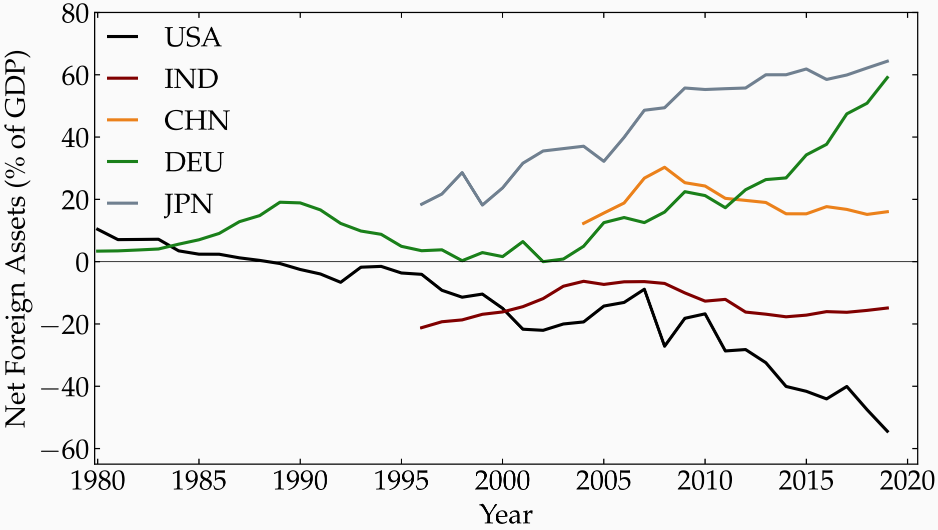
...rates of return on wealth are falling...

Calculation



Source: National Accounts, Flow of Funds, WID.

…and "global imbalances" are rising



Source: International Monetary Fund (IMF), Penn World Table (PWT) 10.1

How will demographics shape these trends in the 21st century?

- Broad agreement that population aging has contributed to historical trends in W/Y, real returns (r), and NFA imbalances
  - Why? An aging population saves more, and aging is uneven across countries

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- Influential view that these trends will revert:

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  "**great demographic reversal**" hypothesis [Goodhart-Pradhan 2020, Mian-Straub-Sufi 2021]

This paper: a sufficient statistic approach to this question

In a baseline multi-country GE overlapping generations (OLG) model, the effect of demographic change on W/Y, r and NFA depends **only** on:

1. Age profiles of wealth, labor income, and consumption
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4. The elasticity of substitution between capital and labor η

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Conclusions are robust to quantitative simulations of richer model

A bridge between reduced-form and structural approaches

- Existing literature follows two broad approaches:

1. **Reduced-form**, based on shift-share exercises
    - Projected asset demand [Poterba 2001, Mankiw-Weil 1989], projected savings rates [Summers-Carroll 1987, Auerbach-Kotlikoff 1990, Mian-Straub-Sufi 2021...]
    - Projected labor supply [Cutler et al 1990], demographic dividend lit. [Bloom-Canning-Sevilla 2003...]
2. **Structural**, based on fully specified GE OLG models
    - Demographics and **wealth** + social security [Auerbach Kotlikoff 1987, İmrohoroğlu-İmrohoroğlu-Joines 1995, De Nardi-İmrohoroğlu-Sargent 2001, Abel 2003, Geanakoplos-Magill-Quinzii 2004, Kitao 2014...]
    - Demographics and **interest rates** [Carvalho-Ferrero-Necchio 2016, Gagnon-Johannsen-Lopez Salido 2016, Eggertsson-Mehrotra-Robbins 2019, Lisack-Sajedi-Thwaites 2017, Jones 2018, Papetti 2019, Rachel-Summers 2019...]
    - Demographics and **capital flows** [Henriksen 2002, Domeij-Flodén 2006, Börsch-Supan-Ludwig-Winter 2006, Krueger-Ludwig 2007, Backus-Cooley -Henriksen 2014, Bárány-Coeurdacier-Guibaud 2019, Sposi 2021...]

- **Sufficient statistic approach** bridges the gap between both

Baseline model

Environment: demographics, production, and government

OLG model, demographic change + multiple countries facing $\{r_t\}$

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Demographics [drop country subscripts]

- Exogenous, time-varying sequence of births N_{0t}
- Exogenous, constant sequence of mortality rates ϕ_j
- No migration

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- Aggregate production fn with capital and effective labor, elasticity of substitution η
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Government

- Flow budget constraint

$$G_t + w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} tr_j + (1 + r_t) B_t = \tau w_t \sum_{j=0}^{T} N_{jt} \mathbb{E} \ell_j + B_{t+1},$$

- Balance budget by changing G_t, not τ_t or tr_{jt}, to keep $B_t / Y_t \equiv \text{cst}$

Environment: heterogeneous agents

Problem for **heterogeneous agents** of cohort k (age $j \equiv t - k$):

$$\max \quad \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right]$$

$$\text{s.t.} \quad c_{jt} + \phi_j a_{j+1,t+1} \leq w_t((1-\tau)\ell(z_{jt}) + tr(z^{jt})) + (1+r_t)a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}\,(1+\gamma)^t$$

- $\sigma \equiv$ elasticity of intertemporal substitution
- β_j: age-specific discount rate
- Φ_j: survival probability by age ($\Phi_j = \prod_j \phi_j$)
- $\ell(z_{jt})$: risky labor supply driven by some stochastic process z_t (Markov chain)
- $\tau, tr(z^{jt})$: taxes and (state-contingent) government transfers
- a_{jt}: annuity holdings

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Equilibrium

Given demographics and policy, in an **integrated world equilibrium**:

- Individuals optimize
- Firms optimize
- Global asset markets clear

$$\sum_c \underbrace{N_t^c \mathbb{E} a_{jt}^c}_{W_t^c} = \sum_c (K_t^c + B_t^c) \quad \forall t$$

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Next: consider small country aging alone, with rest of world at steady state

$\rightarrow$ r constant (will adjust later)

Compositional effects as sufficient statistics

Proposition

The wealth-to-GDP ratio of a small country aging alone with constant r and γ follows

$$\frac{W_t}{Y_t} \propto \frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}$$

where $a_{j0} \equiv \mathbb{E} a_{j,0}$ and $h_{j0} = \mathbb{E} w_0 \ell_{j,0}$ are average initial asset holdings and pretax labor income by age, and $\pi_{jt} = N_{jt}/N_t$ is the share of the population of age j.

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$\Rightarrow$ change in log wealth to GDP ratio:

$$\log\left(\frac{W_t}{Y_t}\right) - \log\left(\frac{W_0}{Y_0}\right) = \log\left(\frac{\sum_j \pi_{jt} a_{j0}}{\sum_j \pi_{jt} h_{j0}}\right) - \log\left(\frac{\sum_j \pi_{j0} a_{j0}}{\sum_j \pi_{j0} h_{j0}}\right) \equiv \Delta_t^{comp}$$

measurable from demographic projections and household surveys

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Why? Demographics do not affect individual decisions, just their aggregation

Measuring compositional effects

Measuring Δ^{comp}

- Use microdata to calculate Δ_t^{comp} for 25 countries:

$$\Delta_t^{comp} \equiv \log\left(\frac{\sum \pi_{jt} a_{jo}}{\sum \pi_{jt} h_{jo}}\right) - \log\left(\frac{\sum \pi_{jo} a_{jo}}{\sum \pi_{jo} h_{jo}}\right)$$

- π_{jt}: projections of age distributions over individuals

  2019 UN World Population Prospects

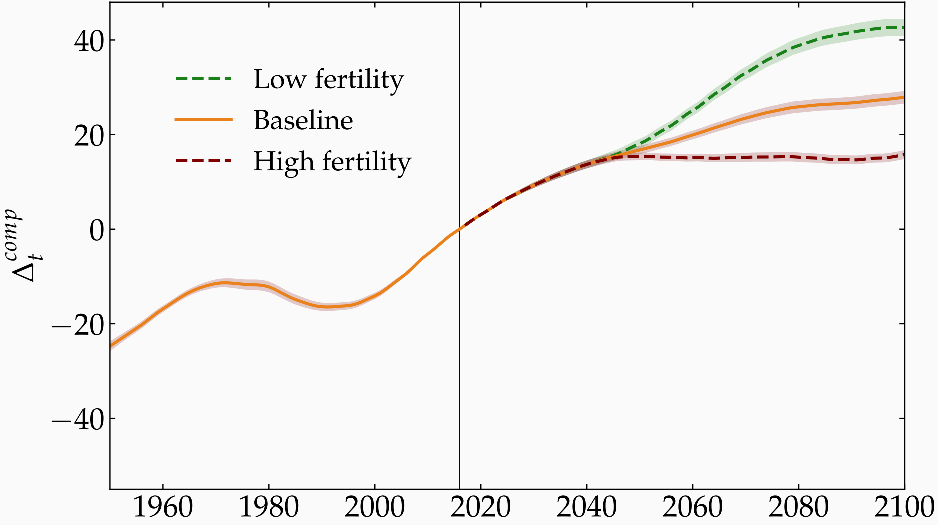
- a_{jo}, h_{jo} age-wealth and labor income profiles in base year ($\equiv$2016)

  For US: SCF, LIS/CPS, and Sabelhaus-Henriques Volz (2019)

  a_{jo} includes funded part of DB pensions

  Household $\rightarrow$ individual (j) by splitting wealth among adults

Δ^{comp} in the United States: 1950-2100



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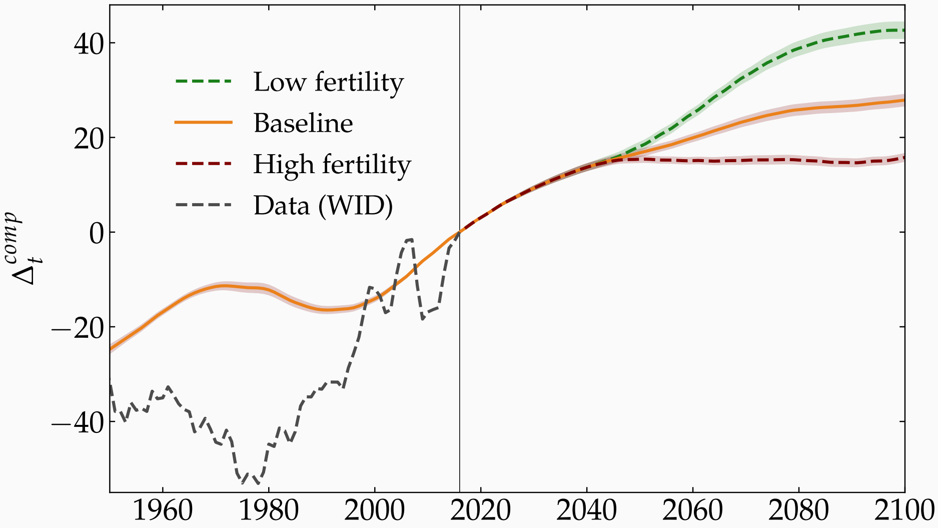
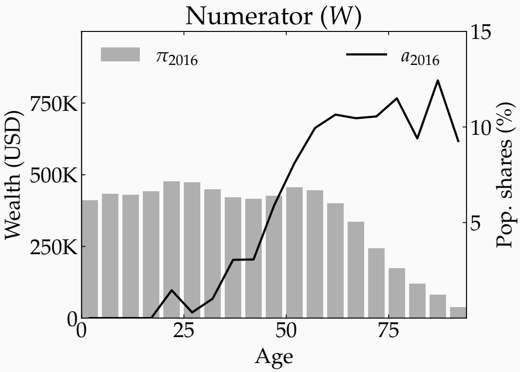


Where do these large effects come from?

▸ Alt. profiles



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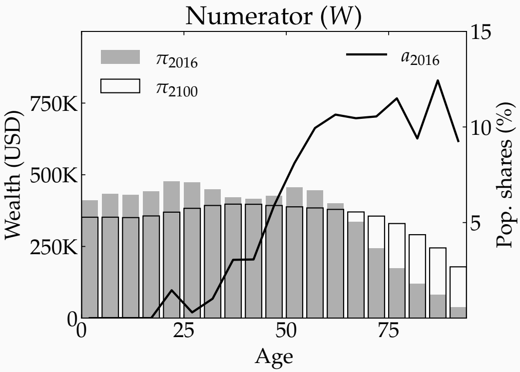

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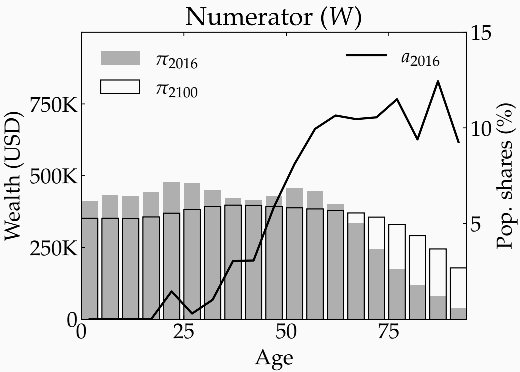

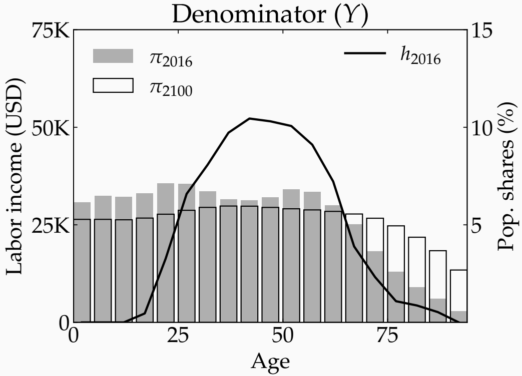

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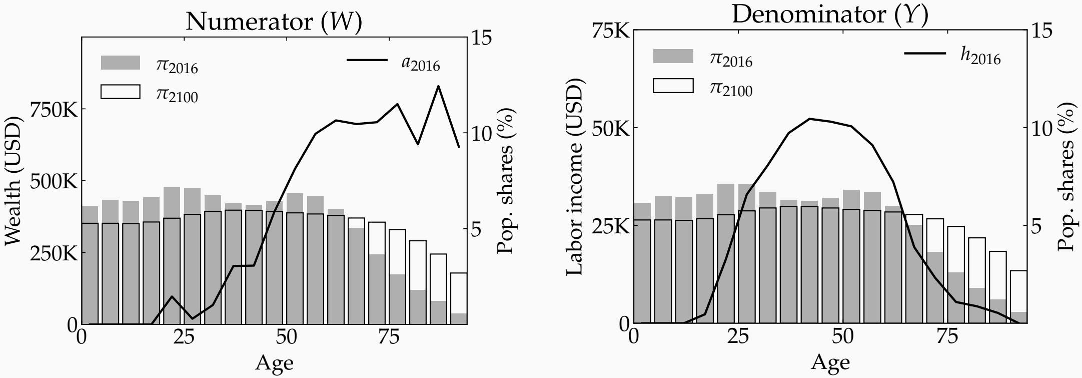
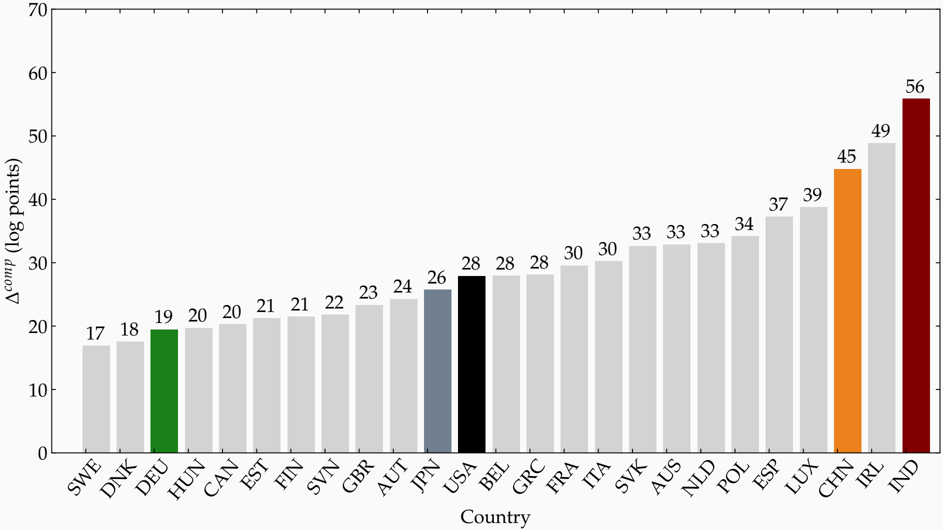


- In paper: separate contribution of numerator and denominator
- Going forward: W contributes $\sim 2/3$, Y contributes $\sim 1/3$
  - Historically demographic dividend pushed Y up, reversed in 2010

Δ^{comp} large and heterogeneous by 2100



General equilibrium implications

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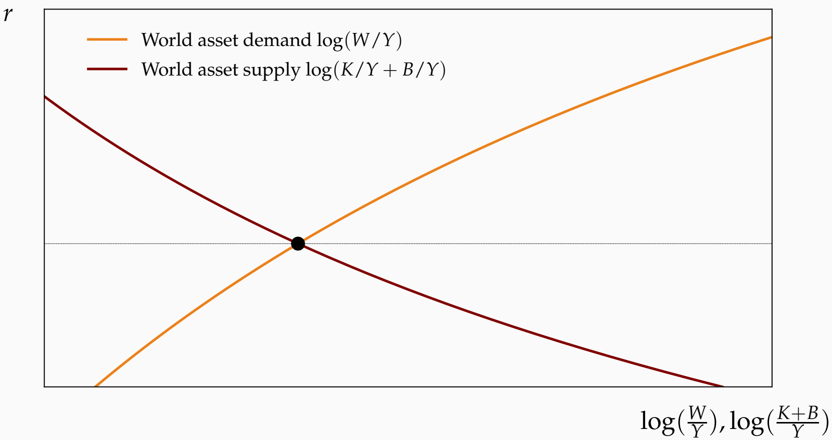
▸ Semielasticity formulas



Semielasticity of asset demand $\bar{\epsilon}_d$: depends on σ, η and observables

Semielasticity of asset supply $\bar{\epsilon}_s$: depends on η and observables

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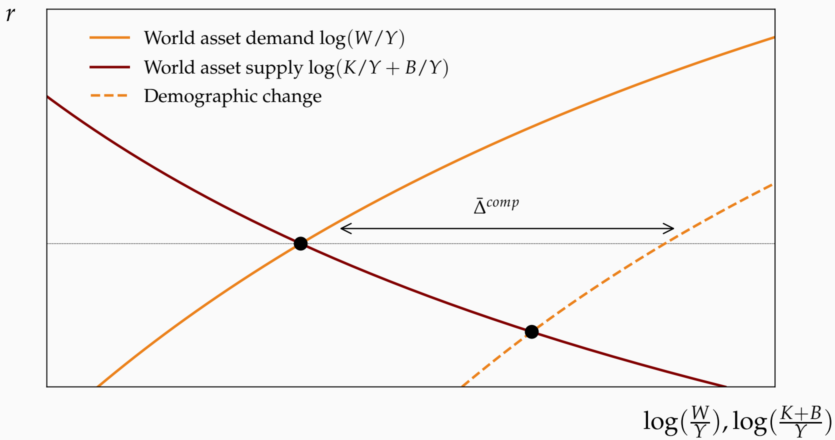
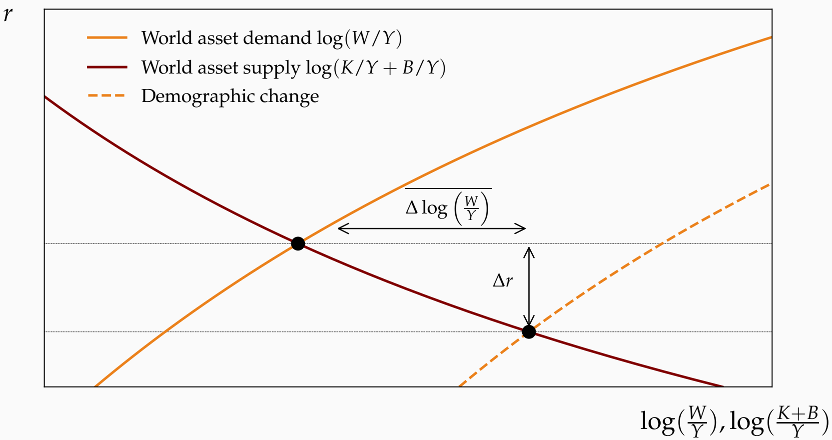


Asset demand shift of $\bar{\Delta}^{comp}$: wealth-weighted average of $\Delta^{comp,c}$

Large and positive in the data.

General equilibrium implications

Semielasticity formulas



$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}_s + \bar{\epsilon}_d} < 0, \quad \overline{\Delta \log\left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}_s}{\bar{\epsilon}_s + \bar{\epsilon}_d} \bar{\Delta}^{comp} > 0$$

Changes in r and W/Y: 2016 to 2100

▸ Risk premium ▸ Historical

$$\Delta r \approx -\frac{\bar{\Delta}^{comp}}{\bar{\epsilon}^d + \bar{\epsilon}^s}$$

$$\overline{\Delta \log\left(\frac{W}{Y}\right)} \approx \frac{\bar{\epsilon}^s}{\bar{\epsilon}^d + \bar{\epsilon}^s}\bar{\Delta}^{comp}$$

A. Change in world r

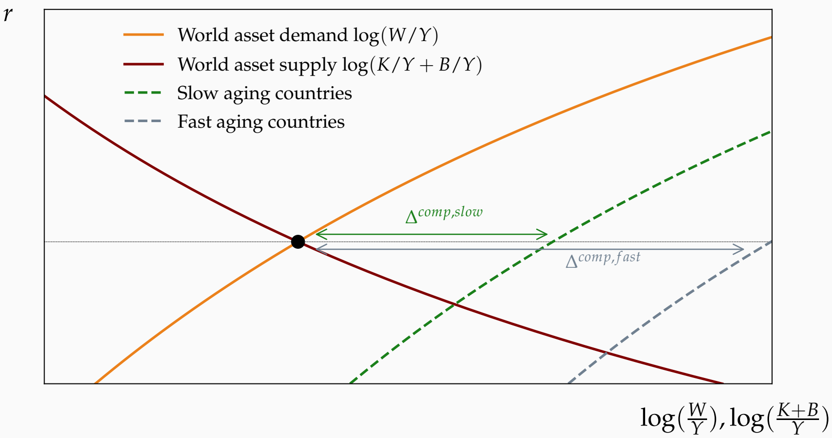
	σ		
η	0.25	0.50	1.00
0.60	-2.45	-1.33	-0.69
1.00	-1.71	**-1.07**	-0.62
1.25	-1.43	-0.96	-0.58

B. Change in avg. $\log W/Y$

	σ		
η	0.25	0.50	1.00
0.60	12.2	6.6	3.4
1.00	14.1	**8.9**	5.1
1.25	14.8	9.9	6.0

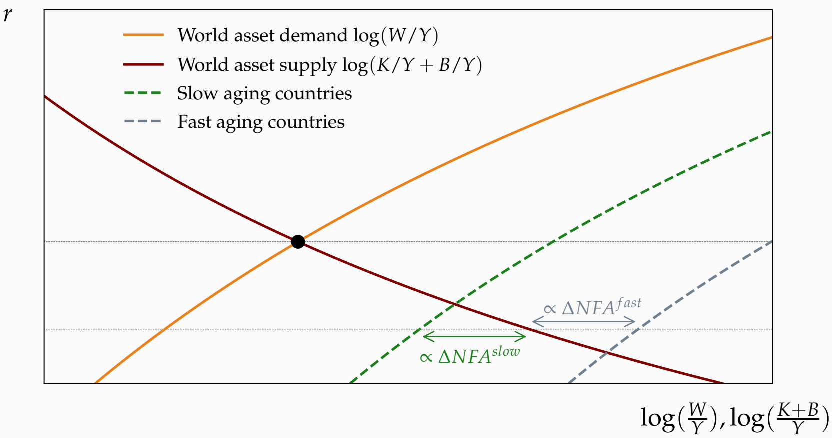
- We obtain very similar outcomes for same σ, η in general model
- Same if we allow demographics to also affect the risk premium
- For 1950-2016, explain 50% of the decline in r and 15% of the increase in W/Y

General equilibrium implications, part 2



Country-specific shifts Δ^{comp} large and heterogeneous in data

General equilibrium implications, part 2

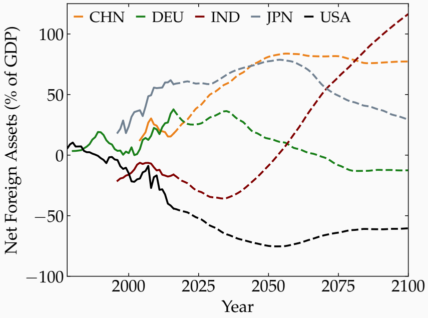


$$\Delta\left(\frac{NFA}{Y}\right) \approx \frac{W_0}{Y_0}\left(\Delta^{comp} - \bar{\Delta}^{comp}\right)$$

Demeaned compositional effect and NFAs

$$\Delta\left(\frac{NFA^c}{Y^c}\right) \simeq \frac{W_0^c}{Y_0^c}\left(\Delta_t^c - \bar{\Delta}_{comp}\right)$$

A. NFA projection



→ Data suggests large global imbalances for the 21st century

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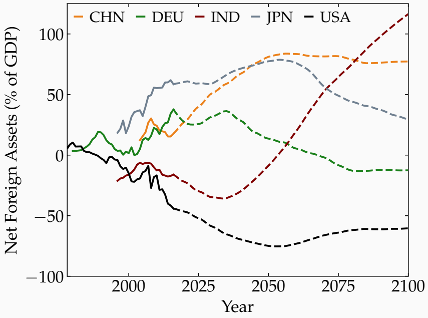
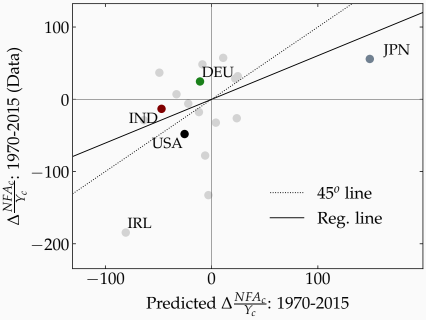


B. Historical performance



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Quantitative model

Updated environment

Household problem becomes (with $\nu \geq \frac{1}{\sigma}$):

$$\max \mathbb{E}_k \sum_j \beta_j \Phi_{jk} \left[\frac{c_{jt}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \Upsilon Z_t^{\nu - \frac{1}{\sigma}} \left(1 - \phi_{jt}\right) \frac{\left(a_{jt}\right)^{1-\nu}}{1-\nu} \right]$$

$$\text{s.t.} \qquad c_{jt} + a_{j+1,t+1} \leq w_t \left((1-\tau_t)\ell_{jt}(z_j)(1-\rho_{jt}) + tr_{jt}(z_j) \right) + (1+r_t) a_{jt} + b_{jt}^r(z_j)$$

$$a_{j+1,t+1} \geq -\bar{a} Z_t$$

- Introducing bequests rather than annuities:
    - assets become bequests at death, distributed as $b_{jt}^r(z_j)$
- Time-variation in mortality Φ_{jk}, labor supply ℓ_{jt}, retirement age ρ_{jt}
- Fiscal rule with adjustments in taxes and transfers
- Income process with intergenerational persistence
- Migration

Robustness of conclusions: steady-state

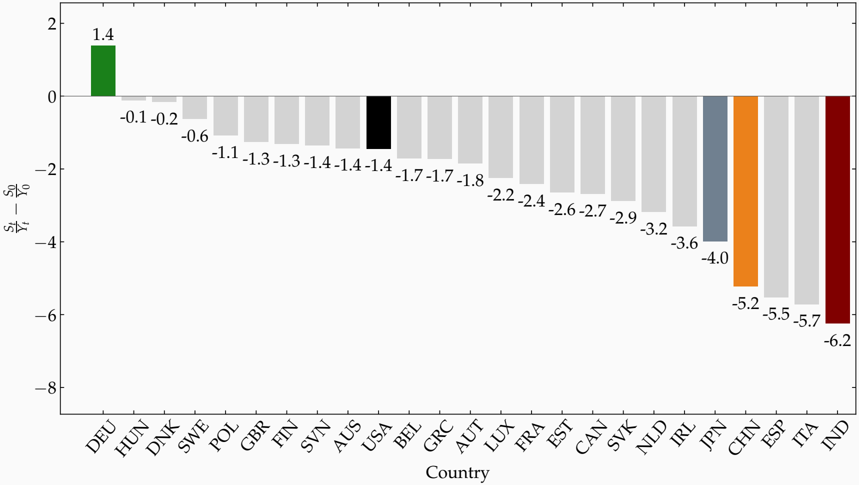
- Assume $\sigma = 0.5$, $\eta = 1$. Let $\overline{\Delta}^{soe} \equiv$ response of W/Y to demographics at fixed r.

	Δr	$\Delta \log \frac{W}{Y}$	$\bar{\Delta}^{comp}$	$\bar{\Delta}^{soe}$	$\bar{\epsilon}^{d}$	$\bar{\epsilon}^{s}$
Extended model	-1.24	12.3	32.2	33.6	19.7	8.3
Sufficient statistic analysis	-1.07	8.9	31.7	31.7	21.2	8.3
From sufficient statistic to extended model						
+ Drop annuities, add bequests	-1.16	12.8	32.1	32.1	17.1	8.3
+ Adjust bequests received	-1.36	15.2	32.1	42.2	23.2	8.3
+ Add income risk	-1.41	15.5	32.2	38.3	18.2	8.3
+ Change perceived mortality	-1.43	14.6	32.2	40.5	20.7	8.3
+ Increase retirement age	-1.25	12.8	32.2	35.0	20.4	8.3
+ Change taxes and transfers (= extended)	-1.24	12.3	32.2	33.6	19.7	8.3
Alternative fiscal rules						
Only lower expenditures	-1.25	12.8	32.2	35.0	20.4	8.3
Only higher taxes	-0.96	8.3	32.2	23.2	17.4	8.3
Only lower benefits	-1.49	15.4	32.2	42.7	20.7	8.3

A great demographic reversal?

Worldwide: decreasing S_t/Y_t everywhere

- Perform same exercise, but projecting S/Y from composition



Declining r despite falling savings?

- Will dissaving of the old reverse the effects of demographics?
  [Lane 2020, Goodhart-Pradhan 2020, Mian-Straub-Sufi 2021, Summers 2023]
- Measured S_t/Y_t from composition does decline
- **But**: r does not increase

Declining r despite falling savings?

- Will dissaving of the old reverse the effects of demographics?
  [Lane 2020, Goodhart-Pradhan 2020, Mian-Straub-Sufi 2021, Summers 2023]
- Measured S_t/Y_t from composition does decline
- **But**: r does not increase
- Why? Savings is misleading with declining pop. growth. In steady state

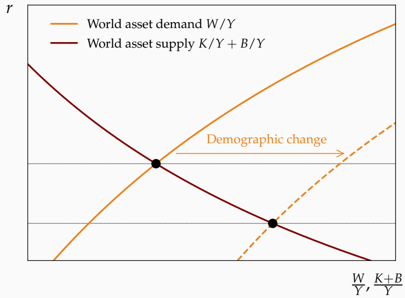
$$\frac{W}{Y} = \frac{S/Y}{g}$$

  where g is GDP growth

- With demographic change, S/Y falls, but g falls by more!

Flows can give the wrong sign for the change in r!

A. Asset demand vs supply



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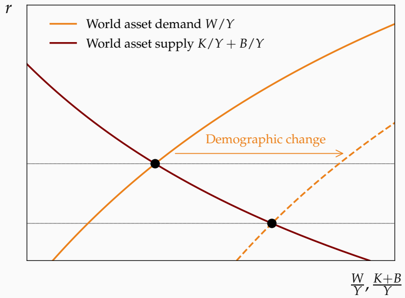
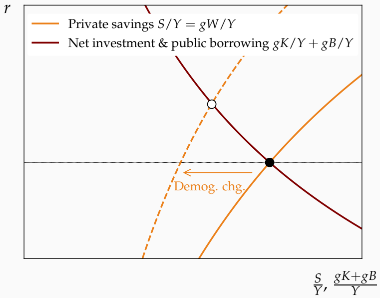


B. Net savings vs investment



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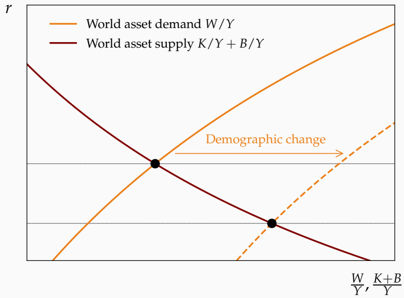

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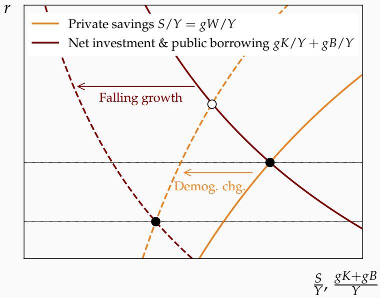


Conclusion

- How do demographics affect wealth-output ratios, real interest rates, capital flows?

  $\rightarrow$ what matters most is the compositional effect Δ^{comp}

  large and **heterogeneous** in the data

- For the 21st century, our approach:
  - Refutes great demographic reversal hypothesis: r definitively falls
  - Suggests the "global savings glut" has just begun

Thank you!

Additional slides

Rates of return on wealth

Back

- Baseline safe return r_t^{safe} is 10 year constant maturity interest rate minus HP-filtered PCE deflator
- Baseline total return is

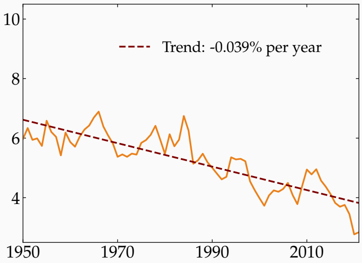
$$r_t = \frac{(s_K Y - \delta K)_t + r_t^{safe} B_t}{W_t - NFA_t}$$

where $(s_K Y - \delta K)_t$ is net capital income

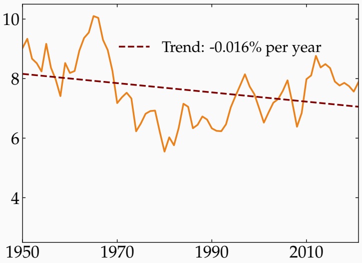
Calculating return: wealth or capital

Back

A. *W* in denominator (baseline)

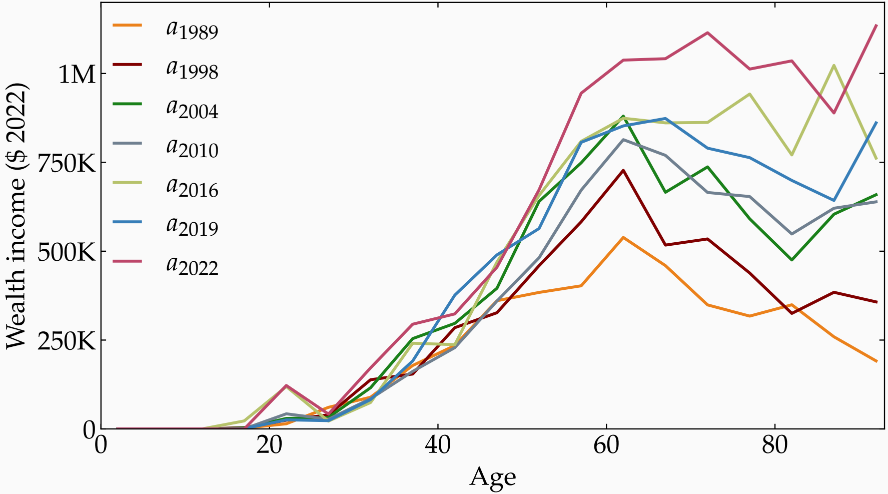


B. *K* in denominator



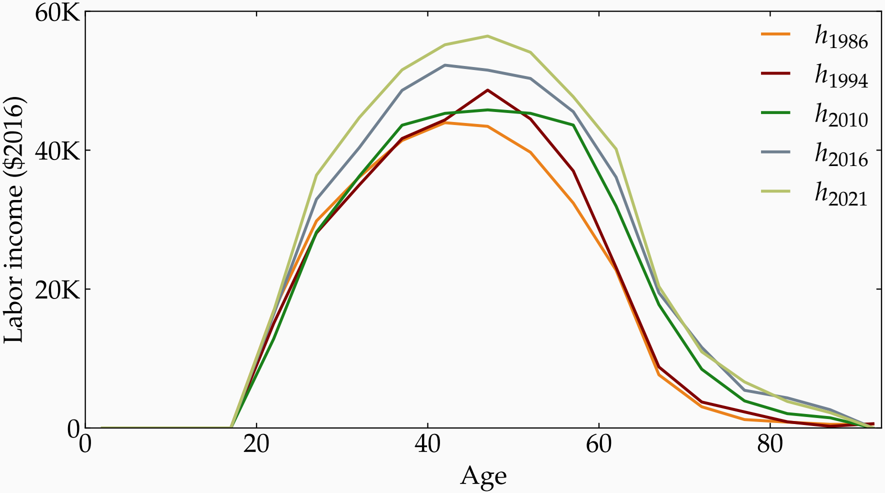
Age-wealth profiles in the U.S.

Back



Age-labor income profiles in the U.S.

Back



Semielasticities of asset demand and supply

◂ Back

- **Asset supply elasticity** $\epsilon^s \equiv \frac{\partial \log(A^s/Y)}{\partial r}$:
  "how will bonds and capital change, relative to GDP, if steady-state r changes?"
  - Given common capital-labor substitution elasticity η, average elasticity is

$$\bar{\epsilon}^s = \frac{\eta}{r_0 + \delta} \overline{\left(\frac{K_0}{W_0}\right)}$$

  $\rightarrow$ Measurable from observables and knowledge of η

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- **Asset demand semielasticity** $\epsilon^d \equiv \frac{\partial \log(W/Y)}{\partial r}$:
  "how will households change average wealth, relative to GDP, if s.s. r changes?"
  - Hard to measure [Saez and Stantcheva 2018: "paucity of empirical estimates"]
  - **Result**: dropping idiosyncratic risk and borrowing constraint from model, **exact formula** for ϵ^d in terms of σ,η, and observables [numerically similar in quantitative model]

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  - In case where $r = g = 0$ and $\eta = 1$, we get:

$$\epsilon^d = \frac{\partial log(W/Y)}{\partial r} = \underbrace{\sigma \frac{C}{W} \text{VarAge}_c}_{\equiv \epsilon^d_{substitution}} + \underbrace{\mathbb{E}Age_c - \mathbb{E}Age_a}_{\equiv \epsilon^d_{income}} \simeq \sigma \cdot 43.7 - 0.6$$

Multiple assets

- Model demand for risky assets: households now solve

$$\max \quad \mathbb{E}_k \left[\sum_j \beta_j \Phi_j \frac{\left(c_{jt} - a_{jt} v_j \left(s_{jt}\right)\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \right]$$

$$\text{s.t.} \quad c_{jt} + \phi_j a_{j+1,t+1} \leq w_t((1-\tau)\ell(z_{jt}) + tr(z^{jt})) + (1 + r_t^f + s_{jt}(r_t^r - r_t^f))a_{jt}$$

$$a_{j+1,t+1} \geq -\underline{a}\left(1+\gamma\right)^t$$

where s_{jt} is risky portfolio share of age j , and $v_j(s_{jt})$ is utility cost of bearing risk

$$v_j^c(s_{jt}) = \overline{r^r - r^f} \cdot (s_{jt} - \bar{s}_j) + \frac{1}{2\Psi}(s_{jt} - \bar{s}_j)^2$$

- New FOC is:

$$s_{jt} = \bar{s}_j + \Psi\left(r_t^r - r_t^f - \overline{r^r - r^f}\right)$$

- Now in addition to aggregate asset demand, must clear market for risky assets

$$\sum_c \sum_j N_{jt}^c \mathbb{E}\left[s_{jt}^c a_{jt}^c\right] = \sum_c K_t^c$$

Sufficent statistics with multiple assets

- Long-run adjustment in asset market:

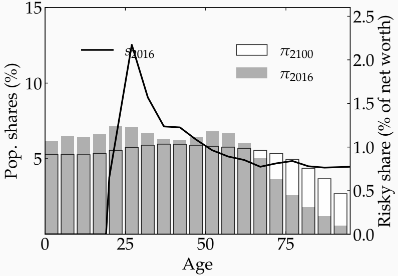
$$\begin{pmatrix} \Delta r \\ \Delta \text{risk premium} \end{pmatrix} = \Sigma \cdot \begin{pmatrix} \Delta^{comp} \log W/Y \\ \Delta^{comp} \text{risky share} \end{pmatrix}$$

- New term: compositional effect on risky share demand $\Delta \text{risky share}$
- Matrix of inverse elasticities Σ affected by Ψ
- Calibrate model as before + matching portfolio shares by age

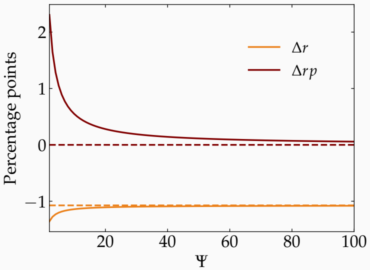
Risky share by age and Δrisky share

Back

A. Risky share vs age distribution

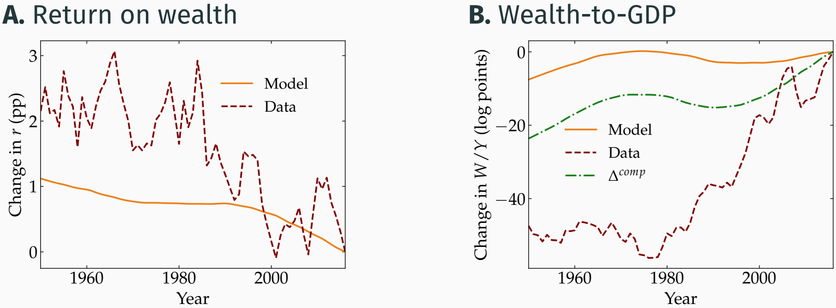


B. Equilibrium change in *r* and *rp*



Back

- Use earlier age distribution to recompute Δ_t^{comp} and ϵ_t^d



- Why? Both asset supply and asset demand shifted up!
  - Demand side: falling TFP growth and rising inequality
  - Supply side: automation, intangible capital, housing, and markups

Demographics accounts for ~**30% of the rise in asset demand**